Electroweak Parity Breaking by Isospin Deformation and the Copenhagen Vacuum*

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Abstract

From the global chiral $SU(2) \times U(1)$ symmetry that appears in the electroweak Left-Right Model in the fundamental representation, a continuous transition to the representation of the Minimal Standard Model is considered in the Cartan subalgebra of the right-handed sector. The connection parameter Δ is the splitting of $U(1)_R$ quantum numbers. Δ is a deformation parameter, breaks $SU(2)_R$ and parity and is proportional to an isomagnetic field, leading to a Copenhagen vacuum structure. A simple mapping on the fundamental representation of $SU(2)_q$ gives Δ in terms of q.

* To appear in the proceedings of 28th Symposion on Elementary Particle Physics, Wendisch-Rietz, August 30 - September 3, 1994

Generalizations of Lie algebras which connect various classical symmetries with one another have been known in mathematics as expanions [1]. In physics they are used to account for deviations from a classical Lie symmetric structure. Such generalizations are particularly useful when generalized generators can be written with an inbuilt classical limit of the Lie algebraic representation. This is the case for the q-deformation $SU(2)_q$ of $SU(2)$ [2]. Strictly speaking, the generalization parameter q is a symmetry breaking parameter. In the Heisenberg spin model for example, q appears as the anisotropy of an XXZ interaction [3]. Deformed spinors are crucial in the construction of momentum vectors with a discrete set of eigenvalues, such that $q \neq 1$ breaks the continuous space-time symmetries in a known and controlled way [4]. The resulting non-commutative geometry is expected to yield finite theories [5, 6] and can therefore be regarded as a new regularisation science, which might make contact with other schemes like e.g. FUT's [7], string theory, where effective uncertainty relations among positions occur [8], or the Connes-Lott version of the Standard Model [9] one day. The latter approaches make definite use of the interplay between internal symmetries and external geometry within the introduction of a fundamental scale parameter. Phenomenologically well motivated electroweak models, where a composite Higgs regulates the Fermi theory and on the other hand is used to look for possible internal symmetries beyond the SM [10], should actually be embedded in such a geometrically more advanced scheme. As a matter of taste, it might nevertheless be taken as an advantage to ask experiment for any possible information first.

In this sense it might be worthwhile to reconsider the breaking of spacetime parity by electroweak interactions using generalized algebras, like it was presented in [11]. To this end we define a transition from the minimal SM with $SU(2)_L \times U(1)_Y$ symmetry to the left-right (LR) model with $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. To be precise, the transition connects the generators of the Cartan subalgebras of the global symmetries. The generalization or transition parameter Δ stands for explicit breaking of parity and (by deformation) of $SU(2)_R$. The fundamental representation of $SU(2)_q$ is, up to a gauge rotation, identical to that one of $SU(2)_\Delta$, giving a very simple expression of Δ in terms of q. Some statements on the special generalized local symmetry are immediate: A $U(1)_{\bar{T}_0}$ subsymmetry of $SU(2)_R$ that is preserved is given by Δ and anomalies are fixed to cancel in the remaining $U(1)_{\bar{T}_0} \times U(1)_{\bar{Y}}$, while the topology of $SU(2)_\Delta$ can be required to be un-

changed from the classical limit and anomalies should neither appear there. The present physical situation is infact just a gauge model in an chromomagnetic, or isomagnetic background, yielding the well-known Copenhagen Higgs Lagrangian [14, 15, 16].

The kinetic and interaction Lagrangian is

$$\mathcal{L}_x = \mathrm{i}\bar{\Psi}_x \gamma_\mu D_x^\mu \Psi_x, \tag{1}$$

where x is summed over L, R; Ψ is an isospinor doublet,

$$D_x^\mu = \partial^\mu + \mathrm{i}g_x \frac{\mathbf{T}_x}{2} \mathbf{W}_x^\mu + \mathrm{i}g' Y_x B^\mu, \tag{2}$$

T_x are the generators of $SU(2)_x$, in the fundamental representation given by Pauli matrices τ_i (normalized to $\tau^2 = 1$) as $T_0 = \frac{1}{2}\tau_3$ and $T_\pm = \frac{1}{\sqrt{2}}(\tau_1 \pm \mathrm{i}\tau_2)$, $\mathbf{W}_x$ the corresponding gauge field triplets and B the $U(1)_Y$ gauge field which couples to vectorlike currents only. We need not consider any spontaneously broken phases, nor family or color degrees of freedom for our purposes here. There is an underlying global chiral symmetry

$$G = SU(2)_L \times SU(2)_R \times U(1)_{Y_L} \times U(1)_{Y_R} \tag{3}$$

and the electric charge operator is defined as

$$Q_x = T_{0x} + Y_x, \tag{4}$$

in each helicity projection of $J_{em} = \bar{\Psi}\gamma^\mu Q \Psi$ such that $Q_L = Q_R = Q = \mathrm{diag}(0, -1)$ [diag $(2/3, -1/3)$] for lepton- [quark-] doublets.

In both the LR and the SM model, $SU(2)_L$ is in the fundamental representation, and

$$Y_L = \frac{B - L}{2}, \tag{5}$$

where $B = 1/6$ is the baryon number and $L = 1/2$ the lepton number. In the LR model [12],

$$\mathbf{T}_L = \mathbf{T}_R, \qquad\qquad Y_L = Y_R. \tag{6}$$

Parity is defined by

$$P : \mathbf{x} = (x_0, x_j) \rightarrow \mathbf{x}' = (x_0, -x_j) \tag{7}$$

for coordinates,

$$\psi(\mathbf{x}) \to \psi'(\mathbf{x}') = \gamma_0 \psi(\mathbf{x}') \tag{8}$$

for spinors and

$$G_\mu(\mathbf{x}) \to G'_\mu(\mathbf{x}') = \epsilon(\mu) G_\mu(\mathbf{x}') \tag{9}$$

for gauge fields G_μ, where $\epsilon(0) = 1$ and $\epsilon(j) = -1$. With the LR assigment eq. (6), the Lagrangian eq. (1) is invariant with respect to P if $g_L = g_R$ and the breakdown will have to occur in low energy states only.

In the SM we have

$$\mathbf{T}_R = 0, \qquad\qquad Y_R = Q \tag{10}$$

instead of the R-numbers in eq. (6). No preserved parity exists for this hidden $SU(2)_R$ [13], P is broken explicitely in the non-abelian as well as in the abelian primordial gauge sector. That nevertheless no breaking term appears explicitely in $\mathcal{L}$ is somewhat unsatisfactory, therefore we make an attempt to set such a term free: Using the splitting of hypercharge quantum numbers of (potential) isospin components, eq. (10),

$$\Delta \equiv \frac{y_R^u - y_R^d}{2}, \tag{11}$$

as a continuous parameter $0 \leq \Delta \leq 1/2$, we can get to eq. (10) from eq. (6) by mapping the generators (Y_R, T_{0R}) of the Cartan subalgebra C_V on generalized quantities

$$\begin{pmatrix} \bar{Y} \\ \bar{T}_0 \end{pmatrix} = \begin{pmatrix} 1 & 2\Delta \\ 0 & 1\text{–}2\Delta \end{pmatrix} \begin{pmatrix} Y_R \\ T_{0R} \end{pmatrix}. \tag{12}$$

For $\Delta \neq 0$, isospin components have different $U(1)$ charges and break

$$SU(2)_R \xrightarrow{\Delta\neq 0} U(1)_{\bar{T}_0} \tag{13}$$

as is seen by redefining $SU(2)_R$ elements $g_R \to \bar{g}_R(\Delta) \equiv (1-2\Delta) g_R$ and probing the generalized hypercharge interaction term to transform like

$$\mathrm{i}\bar{\Psi}_R \gamma^\mu \bar{Y} \Psi_R B_\mu \xrightarrow{\mathbf{T}_R} \mathrm{i}\bar{\Psi}_R \gamma^\mu \bar{Y} \Psi_R B_\mu - \bar{g}_R \bar{\Psi}_R \gamma^\mu [\bar{Y}, T_j \omega^j] \Psi_R B_\mu. \tag{14}$$

The T_0 part in $\bar{Y}$ is nonabelian,

$$[\bar{Y}, T_0] = 0, \qquad\qquad [\bar{Y}, T_\pm] = \pm 2\Delta T_\pm \tag{15}$$

such that the breaking is proportional to $\bar{g}_R \cdot \Delta$, which vanishes in the LR representation at $\Delta = 0$ and in the SM representation at $\Delta = 1/2$, where the action of $SU(2)_R$ becomes trivial. Here, $\bar{Y}$ violates $SU(2)_R$ just like Yukawa couplings $g_{top} \neq g_{bottom}$ violate the custodial $SU(2)$ in the SM scalar sector. The transition eq. (12) preserves eq. (4) and $\bar{Y}$ is always traceless so that no axial anomalies infect the $U(1)_{\bar{T}_0} \times U(1)_{\bar{Y}}$ subsector.

Instead of redefining all $SU(2)_R$ elements it is straightforward to take the map eq. (12) as a deformation on the fundamental representation, following the work of Curtright and Zachos on various kinds of deformations [2]. With $\bar{T}_0$ and $T_\pm$ we have Δ in the commutators,

$$[\bar{T}_0, T_\pm] = \pm(1-2\Delta)\; T_\pm, \qquad [T_+,\, T_-] = (1-2\Delta)^{-1}\bar{T}_0 \tag{16}$$

or, for the real representation,

$$[\bar{T}_i,\, \bar{T}_j] = \mathrm{i}\bar{\epsilon}_{ijk}\bar{T}_k, \tag{17}$$

$$\bar{\epsilon}_{ijk} = \begin{cases} \pm(1-2\Delta)^{-1} & \text{for } i\,(j) = 1\,(2),\; k = 3 \\ \pm(1-2\Delta) & \text{for } i \text{ or } j = 3,\; k = 1 \text{ or } 2 \\ 0 & \text{else} \end{cases} \tag{18}$$

where $\bar{\epsilon}_{ijk}$ becomes the ordinary ϵ_{ijk} when $\Delta \to 0$ and $T_\pm = \frac{1}{\sqrt{2}}(\bar{T}_1 \pm \bar{T}_2)$. The Casimir may be written as

$$C_\Delta = 2\,T_+\,T_- + \bar{T}_0\left[\bar{T}_0 - (1-2\Delta)^{-1}\right] \tag{19}$$

and is minimal at $\Delta = 1/2$. The corresponding classical breaking term is the rescaling of the 3rd direction or anisotropy proportional Δ: For the Lagrangian of a classical free particle, the deforming map eq. (12) gives

$$L \;\to\; L' = \frac{1}{2m}p'^2 = L - U \tag{20}$$

$$U \;=\; -\frac{2\Delta}{m}p_3^2 + ..., \tag{21}$$

where $U = -\mathbf{A}\cdot\mathbf{v}$ contains a vector potential

$$\mathbf{A} \equiv (0,\, 0,\, A_3 = 2\Delta p_3). \tag{22}$$

The breaking term is thus a magnetic field $\mathbf{H} = \nabla \times \mathbf{A}$, which causes the breakdown eq. (13) and we are dealing with some kind of 'Zeeman-deformation'. The SM quantum numbers eq. (10) are reached at $\Delta \to 1/2$, where $\mathbf{A}$ diverges.

Iso- and chromomagnetic backgrounds have been studied as models for the QCD vacuum already in the seventies, because the minimum energy density appears to be at finite $H \sim \Lambda^2$ [14]. An imaginary contribution to the energy density represents a massive harmonic oscillator mode, which is lower in energy than the static field and behaves like a 1+1 dimensional tachyonic Higgs mechanism [15]. Stability conditions have been investigated [16]. A lattice study with anisotropically shifted link variables was done in [18]. Recent results have been reported by K.-J. Biebl and H.-J. Kaiser at the meeting [19]. A condensation enhancement in the 2+1 dimensional Nambu-Jona-Lasinio model in magnetic backgrounds was recently demonstrated [17].

A factor analogous to the ones in the commutation relations eq. (16) and (17) appears in the fundamental representation of the much discussed q-deformation $SU(2)_q$ of $SU(2)$ in the form

$$[\bar{T}_0,\, \bar{T}_\pm] = \pm \bar{T}_\pm, \qquad\qquad [\bar{T}_+, \bar{T}_-] = [\bar{T}_0]_{q^2}, \tag{23}$$

where $[x]_{q^2} \equiv (q^{2x} - q^{-2x}/(q^2 - q^{-2})$.

The solution of eq. (23) $\bar{T}$ in terms of the classical T's is

$$\begin{aligned}
\bar{T}_0 &= T_0, \\
\bar{T}_+ &= \sqrt{\frac{2}{q+1/q} \cdot \frac{[T+T_0]_q\,[1+T-T_0]_q}{(T+T_0)\,(1+T-T_0)}}\, T_+, \\
\bar{T}_- &= \sqrt{\frac{2}{q+1/q} \cdot \frac{[T-T_0]_q\,[1+T+T_0]_q}{(T-T_0)\,(1+T+T_0)}}\, T_-.
\end{aligned} \tag{24}$$

For $T = 1/2$ we get

$$\bar{T}_\pm = \sqrt{2/(q+1/q)}\, T_\pm \tag{25}$$

and H is in the 3rd cartesian direction,

$$[\bar{T}_1,\, \bar{T}_2] = \mathrm{i}\alpha^2 T_3, \quad [T_3,\, \bar{T}_1] = \mathrm{i}\bar{T}_2, \quad [\bar{T}_2,\, T_3] = \mathrm{i}\bar{T}_1, \tag{26}$$

where $\alpha = \sqrt{2/(q+1/q)}$. Rotating $\hat{\mathbf{e}} = R\,\mathbf{e}$,

$$R = \begin{pmatrix} r^2 & -r^2 & -r \\ -r^2 & r^2 & -r \\ r & r & 0 \end{pmatrix}, \tag{27}$$

where $r = \sin(\pi/4)$, reshuffles the factor and after absorbing the potential into the coordinates by $\mathbf{e} \to \mathbf{e}' = (\alpha\, e_1, \alpha\, e_2,\, e_3)$ we get new coordinates

$$\begin{aligned} \hat{e}_1' &= \alpha r^2 e_1 - \alpha r^2 e_2 - r e_3 \\ \hat{e}_2' &= -\alpha r^2 e_1 + \alpha r^2 e_2 - r e_3 \\ \hat{e}_3' &= \alpha r(e_1 + e_2) = \alpha(\hat{e}_1 + \hat{e}_2). \end{aligned} \tag{28}$$

With

$$1 - 2\Delta = \sqrt{\frac{2}{q + 1/q}}, \tag{29}$$

$\hat{e}_3'$ fulfills the central requirement eq. (12). The limits $q \to 1$ and $q \to \infty$ are recovered by $\Delta \to 0$ and $\Delta \to \frac{1}{2}$ in eq. (29). Martin-Delgado has previously interpreted the deformation as an H-field effect [20]. There the symmetry $q \leftrightarrow q^{-1}$ corresponds to a freedom in the sign of H.

All above solutions of commutation relations use finitenes of the representation, i.e. the requirement of highest and lowest weight states to exist for making the difference constant vanish. The latter would otherwise appear as a constraint in classical equations of motion. From eq. (24) as well as in eq. (12) and (16), the generalized generators $\bar{T}_\pm$ create the vacuum like in the classical limit, $\bar{T}_\pm|t, t_0 \pm 1\rangle = 0$, and shall thus preserve the topology and avoid singularities in the functional measure of the generalized action.

Acknowledgement
I wish to thank the organizers for the invitation and very nice conference. For a number of helpful discussions I thank K.-J. Biebl and G. Zoupanos.

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